

Question 9 (12 marks)

$$\text{a. i. } A = 2 \left[(9 \times 9) - (3 \times 1) - \int_3^9 \frac{1}{9} x^2 dx \right] \quad \text{M1}$$

$$= 2 \left[81 - 3 - \left[\frac{x^3}{27} \right]_3^9 \right] \quad \text{M1}$$

$$= 2[78 - (27 - 1)]$$

$$= 2(78 - 26)$$

$$= 104 \text{ cm}^2 \quad \text{A1}$$

$$\text{ii. volume} = 104 \times 18$$

$$= 1872 \text{ cm}^3 \quad \text{A1}$$

$$\text{b. maximum rate when } R'(t) = 0 \quad \text{M1}$$

$$R'(t) = 6t - 4$$

$$= 0$$

$$t = \frac{2}{3} \Rightarrow t = 40 \text{ minutes} \quad \text{A1}$$

$$\text{c. } V(t) = \int R(t) dt \quad \text{M1}$$

$$= \int (3t^2 - 4t + 1) dt$$

$$= t^3 - 2t^2 + t$$

as $V(t) = 0$ at $t = 0$ as no water emptied M1

$$t^3 - 2t^2 + t = 1872 \quad \text{M1}$$

$$(t - 13)(t^2 + 11t + 144) = 0$$

$$\therefore t = 13 \text{ hours} \quad \text{A1}$$

$$\text{d. At } t = 10, V(10) = 10^3 - 2(10)^2 + 10$$

$$= 810 \text{ cm}^3 \quad \text{M1}$$

$$\therefore \text{yes, overflows as only } 810 \text{ cm}^3 \text{ removed} \quad \text{A1}$$